Vita Mathematica

Volume 20

Edited by
Martin Mattmüller

More information about this series at http://www.springer.com/series/4834

Renate Tobies

Felix Klein

Visions for Mathematics, Applications, and Education

Revised by the Author and Translated by Valentine A. Pakis

Birkhäuser

Renate Tobies
Friedrich-Schiller-Universität Jena
Jena, Germany

Translated by
Valentine A. Pakis
Saint Paul, MN, USA

Originally published in German under the title: *Felix Klein: Visionen für Mathematik, Anwendungen und Unterricht*. Berlin: Springer Spektrum, 2019.

ISSN 1013-0330 ISSN 2504-3706 (electronic)
Vita Mathematica
ISBN 978-3-030-75784-7 ISBN 978-3-030-75785-4 (eBook)
https://doi.org/10.1007/978-3-030-75785-4

Book layout: Stefan Tobies
Cover illustration: Felix Klein, 1875, Private Estate Hillebrand, Scheeßel

This book is published under the imprint Birkhäuser, www.birkhauser-science.com by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Figure 1: Felix Klein, 1875 [Hillebrand].

“Whoever shall live on in the memory of the wide world must have had an impact on that world.”

(BLUMENTHAL 1928, p. 2)

PREFACE

Richard Courant spoke euphorically about Felix Klein (1849-1925): "His life was full of intellectual vigor and the will to act, both spurred by a brilliant imagination that was always contriving more and more new designs. He was entirely the sort of wise man and ruler described in Plato's *Republic*."[1] With his *Erlangen Program*, Klein convincingly redefined geometry: geometric properties as invariants of transformation groups. He systematized mathematical theories by recognizing and explaining the interrelations between different disciplines. His visionary programs concerned mathematics and its applications, but also history, philosophy, and pedagogy from kindergarten through higher education. He was extraordinarily engaged, as his admirers would say, in raising awareness for the "eminent cultural significance of mathematics and its applications."[2]

In 1892, the famous Austrian theoretical physicist Ludwig Boltzmann extolled Klein's all-encompassing activity:

> […] Klein's work encompasses almost all areas of mathematics. Especially noteworthy are his contributions to the following areas:
> 1 Algebra and its application to the theory of algebraic forms, number theory, geometry, the resolution of higher equations.
> 2 General theory of functions, theory of elliptic, Abelian, θ-functions and of Riemann surfaces;
> 3 Theory of differential equations;
> 4 Foundations of geometry, curvature and other shape relations of curves and surfaces, also newer geometry and projectivity, the application of geometry to mechanics.[3]

The present book deals with Klein's multifaceted programs and the development of his works. It sheds light on how Klein became a scientist who was able to attract students – male and female alike – to follow his visions.

In 1870, Klein became the first German mathematician to seek personal contact with French mathematicians since Plücker, Dirichlet, and Jacobi had done this some decades before. Klein traveled several times to the British Isles, to Italy, to the United States, etc. He was at the center of the first international congresses of mathematicians and was elected the first chairman of the International Commission on Mathematical Instruction (ICMI) in 1908. In Germany, Felix Klein steered the fortunes of the German Mathematical Society three times as its chairman and, as a professor emeritus, he was still considered the "foreign minister" of mathematics. In the 1890s, the French mathematician Charles Hermite gushingly

1 COURANT 1926, p. 211.
2 [UAG] Math.-Nat. Fak. 25, Valentiner (report from July 19, 1924).
3 Quoted from HÖFLECHNER 1994, pp. 173–74 (Boltzmann to Paul von Groth). – Regarding the context, see Section 6.5.2.

referred to Klein as "a new Joshua in the promised land."[4] Klein became a citizen of the world, explicitly condemning national chauvinism (see Section 8.4).

A precocious student, Klein had completed secondary school at the age of sixteen, earned a doctoral degree at the age of nineteen, and completed his post-doctorate (*Habilitation*) at the age of twenty-one. He was offered his first full professorship at the age of twenty-three, at the University of Erlangen (1872). This was followed by positions at the Polytechnikum in Munich (1875), the University of Leipzig (1880), and the University of Göttingen (as of 1886).

More than just focusing on Klein's professional achievements, this book will also be concerned with Klein as a person. At the age of twenty-six, he married Anna Hegel, the granddaughter of the great philosopher. Her extant letters to Felix Klein document their good relationship and demonstrate that she was often involved in his academic work. Of their four children (one son, three daughters), their son would go on to pursue a technical career. Their youngest daughter studied mathematics, physics, and English in Göttingen and at Bryn Mawr College in the United States. She achieved a distinguished career as a teacher and school principal until 1932; later, she was demoted during the Nazi regime.

Klein cultivated a cooperative working style. At the age of twenty, he found his most important partner in the Norwegian Sophus Lie. Klein wanted to work together, not in competition. Nevertheless, he had to deal with opponents, competitors, different views and interests. David Hilbert, who, on the occasion of Klein's sixtieth birthday in 1909 also invited Henri Poincaré and Gösta Mittag-Leffler to Göttingen, referred in his speech then to Klein's opponents and supporters and expressed his own affinity for Klein.[5]

Klein was not, from the outset, the "Zeus enthroned above the other Olympians," as Max Born experienced him during his own years as a student ("He was known among us as 'the Great Felix'," Born went on, "and he controlled our destinies").[6] We will instead encounter a mathematician who was often plagued by self-doubt and who worried that he might not be able to live up to his own high standards. Early translations of his work and his efforts as the chief editor of the journal *Mathematische Annalen* brought him fame and influence.

With his finger on the pulse of international trends, Klein left a lasting mark on many areas of mathematics, its applications, and organization in Germany. In an astounding number of areas, he was in fact a pioneer.[7] At the University of Göttingen, Klein had laid the foundation for a new golden era and had pointed the way ahead, as Hilbert put it (see Appendix 12). This meant that he appointed the best scientists (among them Hilbert, Hermann Minkowski, Carl Runge, Ludwig Prandtl, Edmund Landau) to work beside him, that he found new ways to retain them in Göttingen, and that he established new institutes by acquiring funds from industry – inspired by the example of American universities.

4 For the context of this quotation, see Section 8.2.2 of this book.
5 See TOBIES 2019b, pp. 513–14, Engl. trans. in ROWE 2018a, pp. 198–99.
6 BORN/BORN 1969, p. 16.
7 For a summary of Klein's pioneering achievements, see Section 10.2.

Right up into old age, Klein was open to new mathematical, scientific, and technical theories. Thus he also identified open problems in the fields of fluid dynamics and statics. In partial collaboration with Emmy Noether, he made significant contributions to the theory of relativity, acknowledged by Albert Einstein.

Klein recognized the specific talents of his students with great foresight. He promoted gifted persons regardless of their religion, nationality, and gender. He guided more than fifty doctoral students, including two women (an Englishwoman and an American) as well as further students from abroad, to new results.

During his lifetime, he received numerous honors, and his versatility is still widely recognized today. Since the year 2000, the European Mathematical Society has awarded a "Felix Klein Prize" to young scientists for outstanding research in applied mathematics (this award was initiated by the Fraunhofer Institute for Industrial Mathematics in Kaiserslautern). Since 2003, moreover, the ICMI has presented a "Felix Klein Award" for lifetime achievements in the field of mathematical pedagogy. In Germany, several institutions have been named after him. There is a Felix Klein Lecture Hall and a Felix Klein Colloquium at the Heinrich Heine University in Düsseldorf (Klein's birthplace) and at the University of Leipzig as well. There is a Felix Klein building at the University of Erlangen and a Felix Klein program at the Technische University in Munich (including a "Felix Klein Teaching Prize"). In Göttingen, there is a secondary school named after Felix Klein, and the meeting room of the Mathematical Institute of the University is adorned by the original Max Liebermann portrait of Klein. The names of the donors who funded this painting are an expression of Klein's worldwide network, which extended as far as India and Japan.[8]

After the late Leipzig historian of mathematics Hans Wußing had encouraged me to study the life and work of Felix Klein, it was the American historian of mathematics David E. Rowe who first enabled me – when Germany was still divided – to study the archival materials pertaining to Klein in Göttingen. The mathematician Helmut Neunzert invited me to give lectures at the University of Kaiserslautern with the following words: "We like to use Klein's arguments to promote the applications of mathematics even today!" A Felix Klein Center was established there in 2008.

Robert Fricke, the mathematician and erstwhile rector of the Technische Hochschule in Braunschweig, aptly compared Felix Klein (an uncle of Fricke's wife) to a triptych, the central panel of which should be devoted to Klein the researcher, while the two flanking panels should depict him as an academic teacher and an outstanding organizer.[9] The goal of this book is to put this triptych into words and enrich it with a human dimension.

Jena, March of 2021 Renate Tobies

8 See Section 8.5.2, and Appendix 10, Fig. 43. – The portrait of Hilbert in the same room was painted in 1928 by Eugen Spiro, who was forced to emigrate in 1935. On Hilbert, see in particular Sections 6.3.7.3 and 7.9 in this book.

9 FRICKE 1919, p. 275. – See the genealogy in Figure 2.

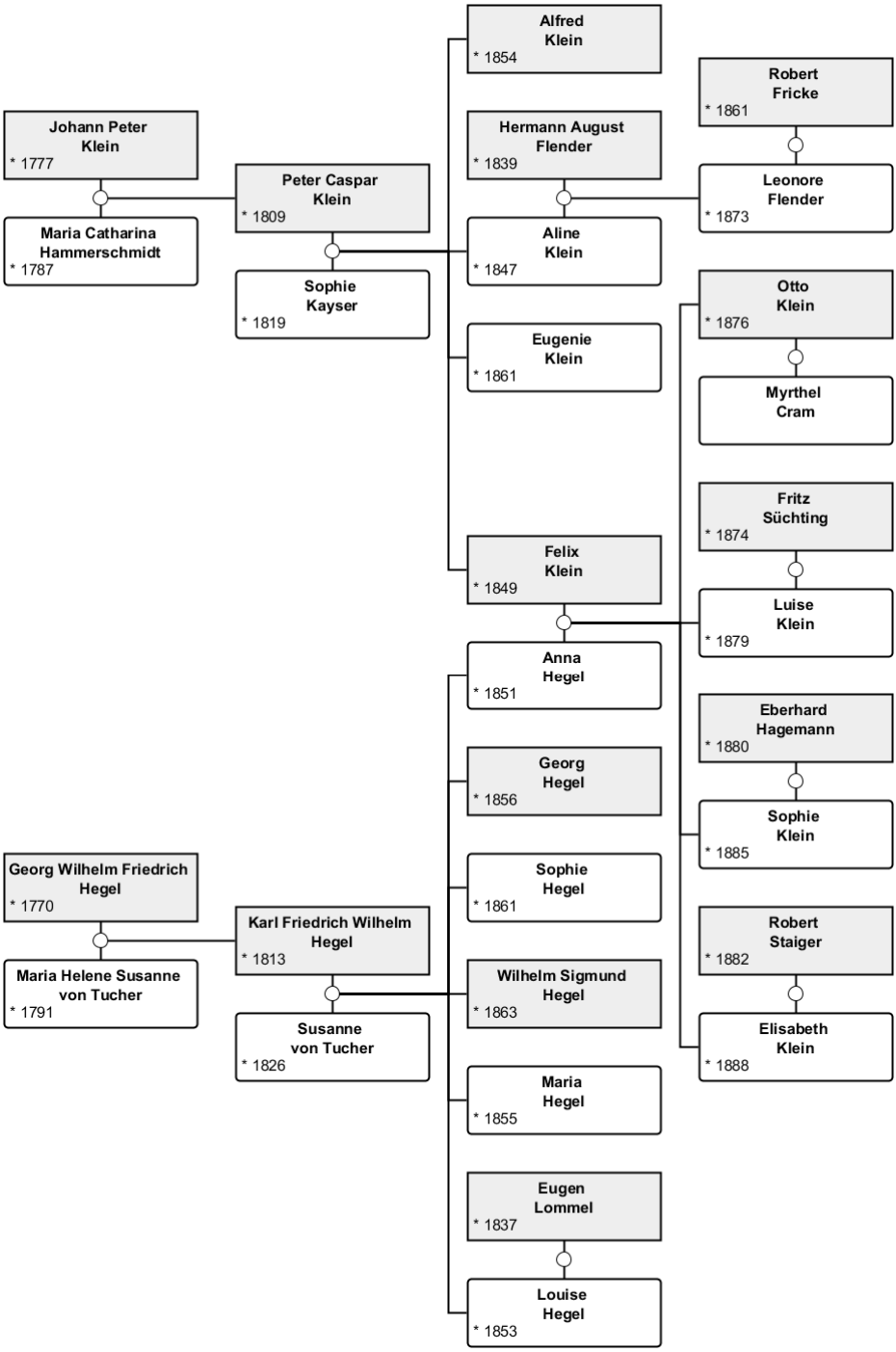

Figure 2: An excerpt from the Klein-Hegel Family Tree (my own design, from [Hillebrand])

CONTENTS

LIST OF TABLES

LIST OF FIGURES